The **kelvin**: new definition, selected mises-en-pratique and direct link with SMD activities

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Etalons Nationaux/Nationale Standaarden

24/05/2018
Dr Ir Miruna Dobre

http://www.smd-ens.be
The kelvin: present and future definition

The kelvin, the unit of thermodynamic temperature, is such that the Boltzmann constant has the exact value
\[ k_B = 1.38065\times10^{-23} \text{ joules per kelvin} \]

The kelvin, the unit of thermodynamic temperature, is the fraction \( 1/273.16 \) of the thermodynamic temperature of the triple point of water.
The two constants \([h,k]\) which occur in the equation for radiative entropy offer the possibility of establishing a system of units for length, mass, time, and temperature which are independent of specific bodies or materials and which necessarily maintain their meaning for all time and for all civilizations*, even those which are extraterrestrial and non-human.

\[
L_s(\lambda, T) = \frac{2hc^2}{\lambda^5} \left( \exp \left( \frac{hc}{\lambda kT} \right) - 1 \right)^{-1}
\]

* Max Planck: Ann. Physik 1, 69-122 (1900)

Units based on fundamental constants: an old idea

Max Planck (Nobel price 1918)
Uncertainties in fixed points: T versus T90

ITS-90, results of key comparisons

thermodynamic
A variety of physical laws to measure energy $k_B T$

- Electrical noise generated by a resistor.
- Pressure of a known amount of gas in a known volume – constant volume gas thermometry
- Limiting low pressure speed of sound in a gas
- Speed distribution of the molecules in a gas
- Dielectric constant of a low pressure gas
- Refractive index of a low pressure gas
- Brightness of a hot surface
- Rayleigh Scattering of light from a gas
**Constant volume gas thermometry**

**Equation of state for ideal gas:**

\[ p \, V = N \, k \, T \]

**Assumptions of model:**
- Point-shaped particles
- Interaction only by elastic scattering

**Real gas (virial expansion):**

\[ pV = NkT \left(1 + \frac{B(T)}{N/V} + \frac{C(T)}{N/V}^2 \right) \]

- \( N \) number of particles
- \( p \) pressure of gas
- \( V \) volume (constant)
- \( k \) Boltzmann constant

**CVGT**

- To be measured:
  - Amount of gas
  - Pressure
  - Volume
How to measure thermodynamic temperature?

\[ \Delta \nu_D = \left[ 2 \frac{kT}{(mc_0^2)} \right]^{1/2} \nu_0 \]

\[ u_0^2 = \gamma \frac{kT}{m} \]

\[ \gamma = \frac{c_p}{c_v} \]

\[ p = kT \varepsilon_0 (\varepsilon_i - 1)/\alpha_0 \]

\[ \langle U^2 \rangle = 4 kT R \Delta \nu \]
Principle of AGT method

standing waves in a resonator

to be measured:

- frequency $\nu_a$
- dimensions via microwaves $\nu_m$ or pyknometry, CMM

Quasi-spheres and microwaves:

M.R. Moldover et al.  
*J. Res. NBS* 93(2), 85-144 (1988)

$$k = \frac{M}{\gamma_0 T_{TPW} N_A c_0^2} \lim_{p \to 0} \left( \frac{\nu_a(p)}{\left< \nu_m(p) \right>} \right)^2$$

http://www.bipm.org
CCT documents
### Uncertainty contributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>$u_R/10^{-6}$</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ g mol$^{-1}$</td>
<td>39.947 816(16)</td>
<td>0.390</td>
<td>30.0%</td>
</tr>
<tr>
<td>$T$ K</td>
<td>273.160 000(99)</td>
<td>0.364</td>
<td>26.1%</td>
</tr>
<tr>
<td>$c_0^2$ m$^2$s$^{-2}$</td>
<td>94756.245(45)</td>
<td>0.470</td>
<td>43.6%</td>
</tr>
<tr>
<td>$R$ J K$^{-1}$ mol$^{-1}$</td>
<td>8.314 478 7 (59)</td>
<td>0.711</td>
<td></td>
</tr>
<tr>
<td>$N_A$ mol$^{-1}$</td>
<td>6.022 141 29 (27) $\times 10^{23}$</td>
<td>0.044</td>
<td>0.4%</td>
</tr>
<tr>
<td>$k_B$ J K$^{-1}$</td>
<td>1.380 651 56 (98) $\times 10^{-23}$</td>
<td>0.712</td>
<td></td>
</tr>
</tbody>
</table>

**Explanation of discrepancy between NPL 2013 and LNE 2011 requires error of $\approx 85$ nm on radius compared with NPL uncertainty estimate of 12 nm**

M. de Podesta, R. Underwood, G. Sutton, P. Morantz, P. Harris, D.F. Mark, F.M. Stuart, G. Vargha, G. Machin

Metrologia **50** 354-376 (2013)

$\Delta k = 2.8$ ppm $\pm 1.4$ ppm
http://rsta.royalsocietypublishing.org/content/374/2064
Boltzmann constant value is a mean of best measurements

The CODATA 2017 values of $h$, $e$, $k$, and $N_A$ for the revision of the SI
D B Newell¹, F Cabiati, J Fischer, K Fujii, S G Karshenboim, H S Margolis, E de Mirandés, P J Mohr, F Nez, K Pachucki, T J Quinn, B N Taylor, M Wang, B M Wood and Z Zhang

¹Metrologia, Volume 55, Number 1
Published 29 January 2018

http://economie.fgov.be
Unbroken chain of calibrations for temperature measurements

- Fixed point cells
  - Hg: -38.8344 °C ± 0.0006 °C
  - H₂O: 0.01 °C ± 0.0001 °C
  - Ga: 29.7646 °C ± 0.0004 °C

SPRT calibration in fixed points

Working standard calibration against SPRT

PRT sensor calibration against working standard

Temperature measurement with PRT sensor

Measurement in lab. and in liquid

-39 °C to 30 °C ± 0.001 °C

-39 °C to 30 °C ± 0.005 °C

-39 °C to 30 °C ± 0.05 °C

25.25 °C ± 0.1 °C
Traceability in temperature measurements after SI change

Short/ medium term

• No consequences in the range – 200 °C to 960 °C
• ITS-90 will not change, defined values of $T_{90}$ for all fixed points
• The estimated value of $T_{TPW}$ remains 273,16 K but it is no more exact;
• The uncertainty associated with the TPW will depend on the uncertainty of $k_B$ determination
• $u_r(TPW) = 3,7\times10^{-6}$ corresponds to 0,1 mK
• The uncertainty in $T_{TPW}$ will increase the uncertainty of all fixed points estimates;
# Impact of new definition on the ITS90 SPRT calibration uncertainties

<table>
<thead>
<tr>
<th>Fixed point</th>
<th>NOW</th>
<th>FUTURE</th>
<th>NOW</th>
<th>FUTURE</th>
<th>% increase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Propagated U of TPW [ mK ]</td>
<td>Propagated U of TPW [ mK ]</td>
<td>Expanded total U of SPRT calibration [ mK ]</td>
<td>Expanded total U of SPRT calibration [ mK ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>k=1</td>
<td>k=1</td>
<td>k=2</td>
<td>k=2</td>
<td></td>
</tr>
<tr>
<td>Zn</td>
<td>0.20</td>
<td>0.36</td>
<td>1.23</td>
<td>1.35</td>
<td>10</td>
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<tr>
<td>Sn</td>
<td>0.15</td>
<td>0.25</td>
<td>0.94</td>
<td>1.02</td>
<td>9</td>
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<tr>
<td>In</td>
<td>0.12</td>
<td>0.20</td>
<td>0.78</td>
<td>0.85</td>
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<tr>
<td>Ga</td>
<td>0.08</td>
<td>0.14</td>
<td>0.32</td>
<td>0.39</td>
<td>21</td>
</tr>
<tr>
<td>Hg</td>
<td>0.06</td>
<td>0.10</td>
<td>0.19</td>
<td>0.25</td>
<td>31</td>
</tr>
<tr>
<td>Ar</td>
<td>0.12</td>
<td>0.20</td>
<td>0.62</td>
<td>0.70</td>
<td>13</td>
</tr>
</tbody>
</table>
The core units of future SI...

We will measure time in multiples of:
\[
\frac{1}{\Delta v^{(133\text{Cs})}_{\text{hfs}}}
\]

We will measure current in multiples of:
\[
e \frac{\Delta v^{(133\text{Cs})}_{\text{hfs}}}{c}
\]

We will measure lengths in multiples of:
\[
\frac{c}{\Delta v^{(133\text{Cs})}_{\text{hfs}}}
\]

We will measure mass in multiples of:
\[
\frac{h\Delta v^{(133\text{Cs})}_{\text{hfs}}}{c^2}
\]

https://www.sif.it/corsi/scuola_fermi/mmxvi#196

M. De Podesta presentation in summerschool 196 of International School of Physics "Enrico Fermi"
And 3 non-core units...

https://www.sif.it/corsi/scuola_fermi/mmxvi#196
M. De Podesta presentation in summerschool 196 of International School of Physics "Enrico Fermi"